

Stochastic finite element analysis of simple hip truss

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Abstract- In this paper, nodal displacements and member stresses in a simple hip truss are determined using stochastic finite element method. The random variables considered are wind loads and cross-sectional dimensions for truss members. In this analysis, the random variations in material properties are ignored. The second order perturbation technique is used for stochastic finite element analysis. The statistical properties of nodal displacements and stresses in truss members are verified by modeling the truss in ANSYS Probabilistic Design System Module and using Monte Carlo simulation algorithm. 5000 simulations were performed in ANSYS. The results obtained through both the methods are in good agreement. The maximum difference in nodal displacements and member stress was found to be 0.09mm and 0.0725% respectively. As stochastic finite element method provides a direct analytic solution for the analysis, it is preferred over the time-consuming simulation procedures.

Keywords: Stochastic, ANSYS PDS, MCS, Stress, Truss

I. INTRODUCTION

In civil engineering design, many design parameters are randomly varying with respect to time and/or space. Hence, it is necessary to consider their statistical characteristics in actual designs. **Jun Zhang and Bruce Ellingwood (1995) [8]** stated that most structural parameters, such as material properties, geometric parameters, and loads, are random in nature. Consider the analysis and design of plane truss.

Trusses are normally used in the construction of building roofing systems and bridges. They provide cost effective solution for large roofing systems. The governing design loads for the roof trusses are mainly due to wind pressure. The Wind is random in nature and hence, it is important to consider its statistical characteristic in load estimation. This is also stated by **Gordon A. Fenton & Nancy Sutherland (2011) [6]**. The roof trusses are fabricated using the hot rolled steel angles and other sections. The mechanical properties of these hot rolled steel sections are also statistical variables. Moreover, the hot rolled steel sections have geometrical imperfections which are also probabilistic in nature. All the above factors will influence the load carrying capacity of the truss members. Further, the characteristic material properties and design loads along with partial safety factors are considered in present design standard. In order to produce a performance-

based design of trusses, the limit state design is not sufficient. Hence, it is necessary to use or develop design and analysis procedure to consider the statistical variations in various design parameters.

George Stefanou (2008) [5] carried out a state-of-the-art review of past and recent developments in the SFEM area and indicated future directions. **José David Arregui-Mena et al. (2014) [7]** stated that the three most commonly used methods include Monte Carlo simulation (MCS), perturbation technique and spectral stochastic finite element method. They have also stated that software packages such as ANSYS, FERUM, CalREL, COSSAN, SFEQ8 etc. can be used to perform analysis based on SFEM. According to **Miguel A. Gutierrez et al. (2014) [11]**, perturbation approach of SFEM consists of expressing the displacement vector as Taylor's series expansion with respect to the random input variables. **G. Falsone et al. (2002) [3]** stated that the advantage in perturbation approach is that computation time is less compared to Monte Carlo Simulations. In systems where uncertainty is low, the results yielded by this technique are accurate. The formulation for 2nd order and nth order perturbation technique was provided by **Marcin Kaminski (2013) [9]**. Analysis of a simple bar element using perturbation technique was illustrated by **Michael Kleiber and Tran Duong Hien (1992) [10]**.

This paper presents the analysis of a simple hip truss using 2nd order perturbation-based SFEM. The truss model is the one used by **Alexis Joseph Ludena (2014) [1]**. Randomness in nodal displacements, forces in truss members is estimated using perturbation technique. Calculations are performed using MATLAB. Stiffness matrix of truss will be generated in MATLAB using code written by **A.J.M. Ferreira (2008) [3]**.

In the present study, the results obtained through SFEM and ANSYS PDS are compared in terms of their accuracy and the time required for computation.

II. PRINCIPLES OF SFEM

SFEM is an extension of the classical deterministic finite element approach to the stochastic framework. The aim is

to provide solution of stochastic problems involving finite elements whose properties are random. The first step in the analysis of uncertain systems is the representation of the input of the system. Input consists of the mechanical and geometric properties as well as the loading of the system. Examples are the Young's modulus, Poisson's ratio, yield stress, area of cross section, wind loads etc. these quantities are described by implementing stochastic processes, the probability distribution and correlation structure which are determined through experiments. This paper makes assumptions regarding the probabilistic characteristics of the uncertain parameters due to lack of relevant experimental data. Even though most of the uncertain parameters in engineering problems are non-Gaussian in nature, this paper assumes Gaussian distribution due to its simplicity. The second step is the assessment of response of the system to the uncertain parameters. This is the most important issue in stochastic finite element analysis and is mainly addressed in this paper. There are two main variants of SFEM – perturbation approach and the spectral stochastic finite element method. In perturbation approach, the response vector is expressed as Taylor's series expansion. Spectral stochastic finite element is ignored in this paper. Monte Carlo simulation is also used for performing stochastic finite element analysis.

III. PERTURBATION TECHNIQUE IN SFEM

As already mentioned, it is the Taylor's series expansion of the stochastic finite element matrix and of the response vector of the system.

A. Expression of random variables

Let there be a function $u(b)$ with b as a random variable. This variable is expressed as Taylor's series expansion as given,

$$u(b) = u^0(b^0) + \varepsilon \frac{du(b)}{db} \Big|_{b=b^0} \Delta b + \dots + \frac{\varepsilon^n}{n!} \frac{d^n u(b)}{db^n} \Big|_{b=b^0} \Delta b^n \quad [1]$$

Here, ε is a small perturbation given to the system. (Value is usually taken as 1) The n^{th} order variation is given by

$$\varepsilon^n \Delta b^n = \varepsilon^n (b - b^0)^n \quad [2]$$

Mean value of $u(b_1, b_2, b_3)$ is given by the following expression. (Here, b_1, b_2, b_3 are random variables)

$$E[u(b_1, b_2, b_3)] = u^0(b_1^0, b_2^0, b_3^0) + \frac{d^2 u(b_1)}{db_1^2} \mu_2 b_1 + \frac{d^2 u(b_2)}{db_2^2} \mu_2 b_2 + \frac{d^2 u(b_3)}{db_3^2} \mu_2 b_3 + \frac{du(b_1)}{db_1} \frac{du(b_2)}{db_2} \text{cov}(b_1, b_2) +$$

$$\frac{du(b_2)}{db_2} \frac{du(b_3)}{db_3} \text{cov}(b_2, b_3) + \frac{du(b_1)}{db_1} \frac{du(b_3)}{db_3} \text{cov}(b_1, b_3) \quad [3]$$

This expression of a random variable, can be applied to any equation and the randomness in general.

B. Application in finite element equations

The general equation for a linear-static time-invariant structural system is given by

$$K \cdot u = F \quad [4]$$

Here, K is the stiffness matrix, u is the displacement vector and F is the load vector.

The expression in [1] is applied for all the random parameters in [4]. u is the response vector and is the unknown entity. In order to obtain the response, 3 equations are derived from which the zeroth order, first order and second order response of the system can be determined. Zeroth order parameters make use of mean values (as done in deterministic approach).

Let K^0 , u^0 , and F^0 are the deterministic stiffness matrix, displacement and load vector respectively. If b is the random variable,

$$K^b = \frac{dK}{db} \text{ and } F^b = \frac{dF}{db} \text{ and so on.} \quad [5]$$

Zeroth order equation is given by

$$K^0 \cdot u^0 = F^0 \quad [6]$$

First order equation is given by

$$K^0 \cdot u^b = F^b - K^b \cdot u^0 \quad [7]$$

Second order equation is given by

$$K^0 \cdot u^{(2)} = (F^{bb} - 2K^b \cdot u^b - K^{bb} \cdot u^0) \text{Var}(b) \quad [8]$$

From [6], [7], and [8], the values of u^0 , u^b , and $u^{(2)}$ are determined respectively.

It is to be noted that

$$u^{(2)} = u^{bb} \cdot \text{Var}(b) \quad [9]$$

For an MDOF system, or a system with multiple random variables, multiple first order equations are obtained which are then expressed in the form of a matrix.

If a and b are two random variables, First order equation becomes

$$K^0 \cdot u^{(1)} = F^{(1)} - K^{(1)} \cdot u^0 \quad [10]$$

Here, $K^{(1)} = [K^a \quad K^b]$

$$\mathbf{u}^{(1)} = \{u^a \quad u^b\}^T$$

Second order equation is given by,

$$\mathbf{K}^0 \cdot \mathbf{u}^{(2)} = \mathbf{F}^{(2)} \cdot \mathbf{S}(\mathbf{a}, \mathbf{b}) - 2\mathbf{K}^{(1)} \cdot \mathbf{S}(\mathbf{a}, \mathbf{b}) \cdot \mathbf{u}^{(1)} - \mathbf{K}^{(2)} \cdot \mathbf{S}(\mathbf{a}, \mathbf{b}) \cdot \mathbf{u}^{(0)} \quad [11]$$

$$\text{Here, } \mathbf{K}^{(2)} = \begin{bmatrix} \mathbf{K}^{aa} & \mathbf{K}^{ab} \\ \mathbf{K}^{ba} & \mathbf{K}^{bb} \end{bmatrix}$$

$\mathbf{S}(\mathbf{a}, \mathbf{b})$ is the covariance matrix given by

$$\begin{bmatrix} \text{Var}(\mathbf{a}) & \text{Cov}(\mathbf{a}, \mathbf{b}) \\ \text{symm.} & \text{Var}(\mathbf{b}) \end{bmatrix}$$

Mean value of \mathbf{u} is given by,

$$\mathbf{E}[\mathbf{u}] = \mathbf{u}^{(0)} + \frac{1}{2} \mathbf{u}^{(2)} \quad [12]$$

Variance of \mathbf{u} is given by,

$$\text{Var}(\mathbf{u}) = \mathbf{u}^{(1)} \cdot \mathbf{S}(\mathbf{a}, \mathbf{b}) \cdot (\mathbf{u}^{(1)})^T \quad [13]$$

Statistical properties of responses such as element stress, force can be determined by applying [1] and [3] in corresponding deterministic finite element equations. For a 2D truss, the stress in an element is given by the following expression.

$$\sigma = \frac{E}{l} \begin{bmatrix} -c & -s & c & s \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \end{bmatrix}$$

Where, σ is the stress in element joining node 1 and 2

E is the modulus of elasticity

l is the length of the element

$u_{1x}, u_{1y}, u_{2x}, u_{2y}$ are nodal displacements along X and Y directions

c and s are direction cosines ($\cos\theta$ and $\sin\theta$ respectively, θ is the angle between line, from node 1 to 2, passing through the axis of the element, and the horizontal)

The 4 components of nodal displacements are random in nature.

$$\mu(\sigma) = \sigma^0$$

$$\text{Var}(\sigma) = \begin{bmatrix} \frac{\partial \sigma}{\partial u_{ix}} & \frac{\partial \sigma}{\partial u_{iy}} & \frac{\partial \sigma}{\partial u_{jx}} & \frac{\partial \sigma}{\partial u_{jy}} \end{bmatrix} \text{Cov}(u_{ix}, u_{iy}, u_{jx}, u_{jy}) \begin{bmatrix} \frac{\partial \sigma}{\partial u_{ix}} & \frac{\partial \sigma}{\partial u_{iy}} & \frac{\partial \sigma}{\partial u_{jx}} & \frac{\partial \sigma}{\partial u_{jy}} \end{bmatrix}^T \quad [14]$$

Force in element i is given by

$$\mathbf{F}_i = \mathbf{A}_i \sigma_i$$

Where, A_i is the area of cross-section, σ_i is the stress in element i

Random variables (in this particular problem) are diameter (d) and stress (σ)

$$\mu(F) = \frac{\partial^2 F}{\partial \sigma^2} \text{Var}(\sigma) + \frac{\partial^2 F}{\partial d^2} \text{Var}(d) + \frac{\partial^2 F}{\partial \sigma \partial d} \text{Cov}(\sigma, d)$$

$$\text{Var}(F) = \begin{bmatrix} \frac{\partial F}{\partial \sigma} & \frac{\partial F}{\partial d} \end{bmatrix} \text{Cov}(\sigma, d) \begin{bmatrix} \frac{\partial F}{\partial \sigma} & \frac{\partial F}{\partial d} \end{bmatrix}^T \quad [15]$$

IV. MONTE-CARLO SIMULATION (MCS)

Monte Carlo simulation technique is one of the simplest method to consider the random variation of the design parameters. It has the capability to handle large number of random variables. One of the main tasks in MCS is the generation of random numbers from prescribed probability distributions. For a given set of random numbers, the given process is deterministic.

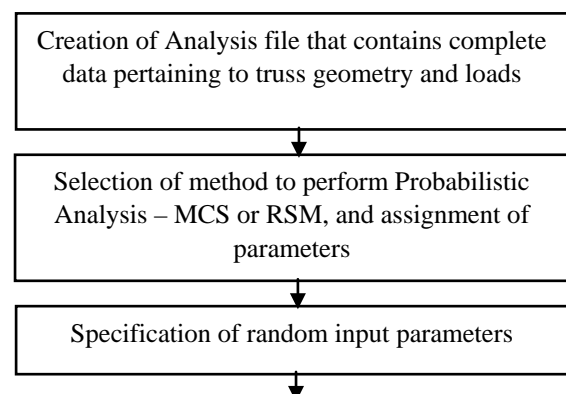
V. ANSYS PROBABILISTIC DESIGN SYSTEM (PDS)

Chunxue Song et al. (2014) [2] carried out a study on the accuracy of ANSYS Probabilistic Design System (PDS), which is provided in ANSYS APDL package. They stated that there are two methods for performing probabilistic analysis in ANSYS PDS – Monte Carlo simulation (MCS) and Response surface method (RSM). They concluded their work stating that both MCS and RSM are accurate and that for problems involving non-linearity, MCS is more accurate.

An important feature in ANSYS APDL is the ability to interact with the algorithm through third party applications. Hence the output can be interpreted in whichever way required. The procedure to perform probabilistic analysis using ANSYS PDS is depicted in the flowchart in section VI.

VI. ANALYSIS USING ANSYS PDS

The entire process of performing probabilistic analysis using ANSYS PDS can be summarized in the flowchart below.



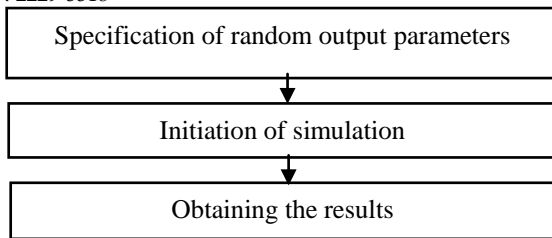


Fig. 1 – Flowchart depicting analysis procedure using ANSYS PDS

VII. ANALYSIS USING SFEM

The entire process of performing probabilistic analysis using SFEM can be summarized in the flowchart below.

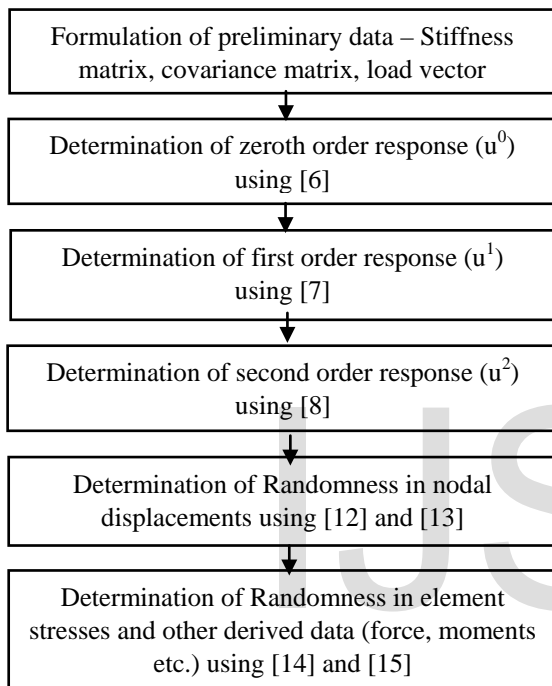


Fig. 2 – Flowchart depicting analysis procedure using SFEM

VIII. CASE STUDY ON HIP TRUSS

Truss model [1], considered for study is shown in Fig. 3. It is an 8-noded 13-bar hip truss with all truss members having uniform circular cross-section. Load on truss and diameter of the truss members are considered to be random variables. Length of truss members, support conditions are all given in Fig. 3

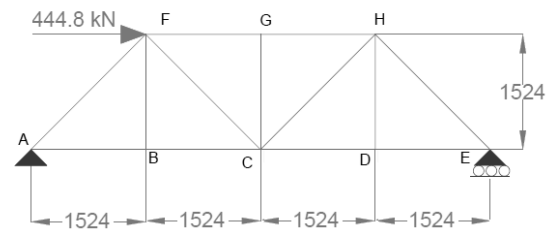


Fig. 3 – Simple hip truss model

The objective of the problem is to find the statistical properties of nodal displacements and stress in truss members. Randomness of load and diameter of truss elements are tabulated as shown below:

Table 1 – Statistical properties of random input parameters

Parameter	Mean	Standard Deviation	Type
P	444.8 kN	44.8 kN	Normal
d	50.8 mm	5.08 mm	Normal

For establishment of reference, the truss is first modelled in ANSYS.

The deformed shape of the truss is shown in Fig. 4

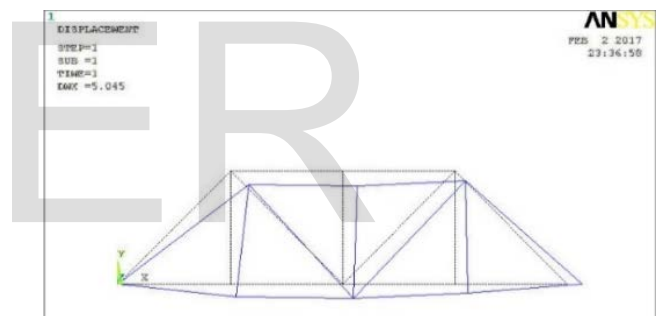


Fig. 4 – Deformed shape of truss

The stress in various truss members is obtained through simple linear static analysis.

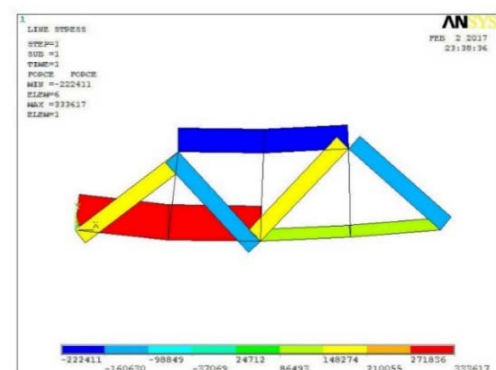


Fig. 5 – Forces in truss members

Probabilistic analysis is performed using the PDS module as per procedure given in Fig. 1. 5000 simulations are performed. Randomness in the input parameters – load P and diameter of truss members are defined in ANSYS PDS Module. The statistical properties of nodal displacements, stresses and forces in truss members are obtained as output. The probability density function (PDF) and cumulative distribution function (CDF) of various random variables are plotted on a graph by ANSYS PDS module based on the sample values. Hence, higher the number of simulations, greater will be the accuracy.

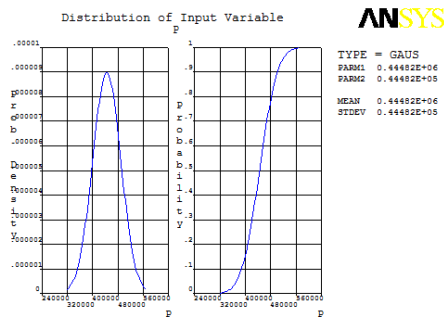


Fig. 6a – PDF and CDF of load applied on truss

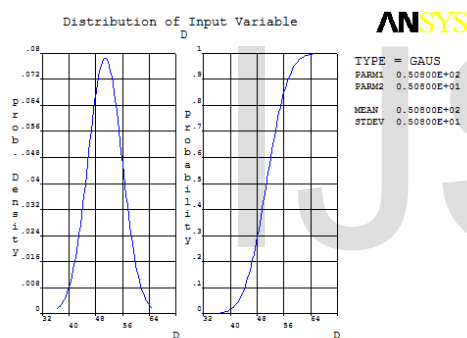


Fig. 6b – PDF and CDF of diameter of truss members

The randomness in force in member 1 (AB):

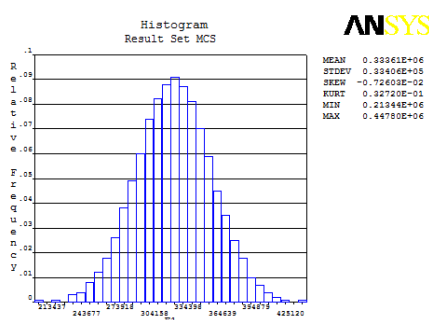


Fig. 7 – histogram depicting frequency of random values for force in element 1 (AB)

IX. RESULTS AND DISCUSSION

A. RESULTS OBTAINED FROM ANSYS PDS

The statistical properties of nodal displacements and member stresses are tabulated in tables 2 and 3 respectively. The entire process took 3098 seconds.

Table 2 – Statistical properties for nodal displacements (obtained from ANSYS PDS)

Node No.	u_x		u_y	
	Mean	SD	Mean	SD
1 (A)	0	0	0	0
2 (B)	1.294	0.3069	-3.020	0.7162
3 (C)	2.588	0.6139	-3.451	0.8185
4 (D)	3.020	0.7162	-2.157	0.5116
5 (E)	3.451	0.8185	0	0
6 (F)	4.240	1.006	-3.020	0.7162
7 (G)	3.377	0.8010	-3.451	0.8185
8 (H)	2.514	0.5963	-2.157	0.5116

Table 3 – Statistical properties of stress in truss members

Element No.	Stress (N/mm ²)	
	Mean	Standard Deviation
1 (AB)	169.8	40.28
2 (BC)	169.8	40.28
3 (CD)	56.61	13.43
4 (DE)	56.61	13.43
5 (EH)	-80.06	18.99
6 (HG)	-113.2	26.85
7 (GF)	-113.2	26.85
8 (FA)	80.06	18.99
9 (FB)	0	0
10 (FC)	-80.06	18.99
11 (GC)	0	0
12 (CH)	80.06	18.99
13 (HD)	0	0

B. Results from SFEM

The procedure as given in Fig. 2 is followed in determination of statistical properties of response of the truss.

Table 4 – statistical properties of nodal displacements obtained through SFEM

Node No.	u_x		u_y	
	Mean	SD	Mean	SD
1 (A)	0	0	0	0
2 (B)	1.267	0.125	-2.956	0.293

3 (C)	2.534	0.251	-3.378	0.334
4 (D)	2.956	0.293	-2.111	0.209
5 (E)	3.378	0.334	0	0
6 (F)	4.150	0.411	-2.956	0.293
7 (G)	3.306	0.327	-3.378	0.334
8 (H)	2.461	0.244	-2.111	0.209

13 (HD)	N/A
Max	0.0725%

Table 5 – statistical properties of member stress obtained through SFEM

Element No.	Mean (N/mm ²)	SD (N/mm ²)
1 (AB)	169.8163	40.2753
2 (BC)	169.8163	40.2753
3 (CD)	56.6929	12.376
4 (DE)	56.5617	12.376
5 (EH)	-80.05	24.43
6 (HG)	-113.255	31.16
7 (GF)	-113.255	31.16
8 (FA)	80.05	24.43
9 (FB)	0	0
10 (FC)	-80.1181	27.53
11 (GC)	0	0
12 (CH)	80.05	24.43
13 (HD)	0	0

The entire process took **3.52 seconds** in MATLAB.

C. Comparison of results

The difference in values obtained through SFEM and ANSYS PDS is shown in table 6.

Table 6 – Variation between SFEM and ANSYS PDS

Node No.	Difference in nodal displacements		Element No.	% Difference in member stress
	u _x	u _y		
1 (A)	N/A	N/A	1 (AB)	0.009
2 (B)	0.027	0.064	2 (BC)	0.009
3 (C)	0.054	0.073	3 (CD)	0.1462
4 (D)	0.064	0.046	4 (DE)	0.1462
5 (E)	0.073	N/A	5 (EH)	0.0124
6 (F)	0.09	0.064	6 (HG)	0.048
7 (G)	0.071	0.073	7 (GF)	0.048
8 (H)	0.053	0.046	8 (FA)	0.0124
Max	0.09 mm		9 (FB)	N/A
			10 (FC)	0.0725
			11 (GC)	N/A
			12 (CH)	0.0124

1. It can be observed that there is a maximum difference in nodal displacement of 0.09 mm in node no. 6 in the values obtained through MCS and SFEM. For member stresses the maximum % difference is 0.0725% for member FC.
2. The computation time required for performing 5000 simulations using ANSYS PDS is 3098 seconds whereas the analysis is performed considering same statistical characteristics for random variable in SFEM takes only 3.52 seconds.

X. CONCLUSION

1. The output responses for a hip truss are calculated using SFEM and the same are compared with the results obtained from MCS in ANSYS PDS.
2. The structural responses obtained through both the methods are in good agreement.
3. The computation time required for ANSYS PDS simulation is quite higher than the SFEM computational time. Further, it is concluded that the perturbation is providing an accurate and efficient solution for structural analysis.

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